

Echoes of Inflationary Particle Phase Transitions in the CMB

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Cosmological phase transitions (CPTs), such as the Grand Unified Theory (GUT) and the electroweak (EW) ones, play a significant role in both particle physics and cosmology. In this letter, we propose to probe the first-order CPTs, by detecting gravitational waves (GWs) which are generated during the phase transitions through the cosmic microwave background (CMB). If happened around the inflation era, the first-order CPTs may yield low-frequency GWs due to bubble dynamics, leaving imprints on the CMB. In contrast to the nearly scale-invariant primordial GWs caused by vacuum fluctuation, these bubble-generated GWs are scale dependent and have non-trivial B-mode spectra. If decoupled from inflaton, the EWPT during inflation may serve as a mirror image of the one after reheating where the baryon asymmetry could be generated via EW baryogenesis (EWBG). The CMB thus provides a potential way to test the feasibility of the EWBG, complementary to the collider measurements of Higgs potential and the direct detection of GWs generated during EWPT.

Introduction. Phase transitions in particle physics may have deep implications in cosmology. One famous example is the invention of inflation theory, which was originally motivated by addressing the missing magnetic monopole problem produced during the GUT phase transition [1]. Another example is related to the puzzle of cosmic baryon asymmetry. If the electroweak phase transition (EWPT) is of first order, the baryon asymmetry could be generated during the phase transition, with CP-violating Higgs couplings [2]. Probing the phase transitions occurred in the early Universe therefore plays a significant role in both particle physics and cosmology.

With the discovery of the Higgs particle, the questions about the EWPT have intensified. Though highly challenging, we expect to be able to probe the first-order EWPT by measuring Higgs self-interaction at High-Luminosity LHC or at future colliders (see, e.g., [3–10]). More generally, the cosmological phase transitions (CPTs) of first order are implemented via bubble nucleation. The expanding bubbles may collide with each other or stir up turbulence in the thermal plasma, yielding gravitational waves (GWs) in spacetime [11]. Particularly, if the phase transitions are of EW scale or PeV scale and happened after reheating [12–16], the produced GWs are characterized by a frequency $\gtrsim 10^{-4}\text{Hz}$ [17] that direct detection experiments, like Advanced LIGO [18], Advanced Virgo [19] and LISA [20], are currently looking for or will look for. In this letter, we propose a new approach of probing the first-order CPTs, by detecting the bubble-generated GWs through the cosmic microwave background (CMB).

The CMB fluctuations in temperature and polarization provide us rich information of the primordial universe. Those fluctuations are measured by experiments like Planck [21] and BICEP2/Keck [22]. Inflation is the leading paradigm in cosmology to provide the seed for those CMB fluctuations, whereas the potential role of the

CMB in probing the first-order CPTs was ignored. One reason could be: if the CPTs happened after reheating, the produced GWs have a characteristic frequency far beyond the scope of CMB.

One fact often ignored before about inflation is that an inflationary universe may undergo some thermal phase transition due to the temperature decreasing. If its characteristic energy scale is above the inflationary Hubble scale and the Higgs field involved only couples to the inflaton gravitationally, such a phase transition is expected to happen during inflation. Here we would stress that the second requirement is not necessary. In some physical contexts, such as the GUT and the recently proposed cosmological relaxation models aimed to solve the hierarchy problem [23], such a scenario for phase transitions could be generic. These phase transitions, if they are of first order, may lead to entirely different cosmological consequences. Particularly, they are different from old inflation scenarios, e.g. [1, 24], where inflation was driven by a first-order phase transition and thus the phase transition can not finish, known as the graceful exit problem. In our scenarios, inflation occurred inside and outside the bubbles simultaneously, enabling the sub-horizon bubbles to collide with each other and hence generate GWs during inflation. Since the phase transitions happened during inflation, the produced GWs can be characterized by scales comparable to the size of the current universe. Particularly, the GWs produced by first-order CPTs have a scale-dependent power spectrum as we will show later. Thus they can be observed in the CMB potentially.

More explicitly, the temperature contribution from radiation drops quickly at the beginning of inflation, left with only a contribution from curvature, known as the Gibbons-Hawking temperature $T_{GH} = H/(2\pi)$ [25]. T_{GH} can vary between 10^{14}GeV to 10^{-24}GeV (see Fig. 1). The vast span of the unknown energy scale of inflation may encompass rich physics. Among different models of infla-

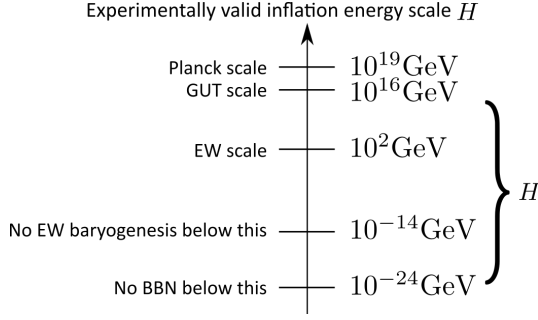


FIG. 1. In various inflation models, Hubble constant during inflation can take different values from 10^{-24} GeV up to 10^{14} GeV. The upper bound is set by the latest experiment [22]. When the Hubble constant is below 10^{-14} GeV, the universe can not reheat above 100 GeV later, where the EW baryogenesis can be hardly achieved. Below 10^{-24} GeV, the reheated universe is too cool to have big bang nucleosynthesis (for a recent bound after *Planck* 2015, see [28]).

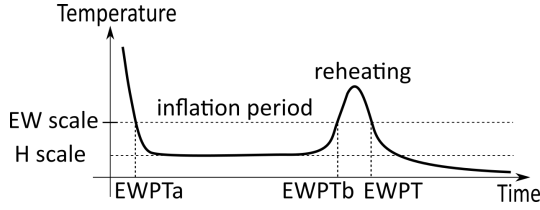


FIG. 2. An exemplifying thermal temperature versus time relation in the early universe. After inflation reaches the attractor phase, the temperature during inflation comes mainly from the curvature contribution of the de-Sitter space, which is comparable with the Hubble constant. Two more electroweak phase transitions are proposed here, namely EWPTa during inflation, and EWPTb when the universe gets heated up during reheating, with 10^{-14} GeV $< H < 10^2$ GeV assumed.

tion with low energy scales [26, 27]: $H \sim T_{GH} \leq 10^2$ GeV, the EWPT will happen during the drastic decreasing of temperature at the beginning of inflation (see Fig. 2). Note, for $H \sim 10^2$ GeV, the energy density is still as high as $\rho \sim (10^{10} \text{ GeV})^4$, ensuring that inflation can happen. Moreover, the GUT phase transition, if exists, will happen in almost all the inflation scenarios.

Gravitational wave spectra. We derive the power spectrum P_γ coming from scale dependent GWs produced in de-Sitter space here. During inflation, the action of GWs takes the form

$$S = \frac{M_p^2}{8} \int d\tau d^3\mathbf{x} a^2 \left[(h'_{ij})^2 - (\nabla h_{ij})^2 \right], \quad (1)$$

where the prime denotes the derivative with respect to conformal time τ and $a(\tau) \approx -1/H\tau$ is the scale factor for quasi de-Sitter space. The GWs have two physical modes. We introduce the polarization tensors ϵ_{ij}^+ , ϵ_{ij}^\times and

decompose the gravitational fields:

$$h_{ij}(\mathbf{k}) = \frac{\sqrt{2}}{M_p} \left[\gamma_+(\mathbf{k}) \epsilon_{ij}^+(\mathbf{k}) + \gamma_\times(\mathbf{k}) \epsilon_{ij}^\times(\mathbf{k}) \right], \quad (2)$$

then the action can be written as two independent copies of probe field γ_s

$$S = \sum_{s=+, \times} \int dt \frac{d^3\mathbf{k}}{(2\pi)^3} a^3 \left[\frac{1}{2} \dot{\gamma}_s(\mathbf{k}) \dot{\gamma}_s(-\mathbf{k}) - \frac{k^2}{2a^2} \gamma_s(\mathbf{k}) \gamma_s(-\mathbf{k}) \right]. \quad (3)$$

We can then quantize the γ_s fields: $\gamma_s(\mathbf{k}, \tau) = v_k(\tau) a_{\mathbf{k}s} + v_k^*(\tau) a_{-\mathbf{k}s}^\dagger$, where $a_{\mathbf{k}s}$ and $a_{-\mathbf{k}s}^\dagger$ are creation and annihilation operators and satisfy the usual commutation relation. Solving the equation of motion, we can get the mode function v_k :

$$v_k(\tau) = \frac{H}{\sqrt{2k^3}} \left(c_1(k)(1+ik\tau)e^{-ik\tau} + c_2(k)(1-ik\tau)e^{ik\tau} \right), \quad (4)$$

where the coefficients $c_1(k)$, $c_2(k)$ are subject to the consistency condition of quantization $|c_1|^2 - |c_2|^2 = 1$.

The energy density of GWs is essentially the Hamiltonian density, given by

$$\rho_{GW} = \int \frac{dk}{k} \frac{k^3}{2\pi^2} \left(|\dot{v}_k|^2 + \frac{k^2}{a^2} |v_k|^2 \right). \quad (5)$$

Furthermore, the gravitational energy spectrum can be obtained as

$$\Omega_{GW}(k, \tau) = k \frac{d\rho_{GW}}{dk} / \rho_{\text{tot}} \quad (6)$$

$$= \frac{1}{3H^2 M_p^2} \frac{k^3}{2\pi^2} \frac{|v'_k(\tau)|^2 + k^2 |v_k(\tau)|^2}{a(\tau)^2}, \quad (7)$$

where we assume that the universe is spatially flat, meaning that $\rho_{\text{tot}} = \rho_{\text{critical}} = 3H^2 M_p^2$. Particularly note that during inflation $\rho_{\text{tot}} = \rho_{\text{inflaton}} + \rho_{\text{rad}} + \rho_{\text{higgs}}$.

We can also calculate the power spectrum of GWs which measures the two point correlation:

$$P_\gamma(k, \tau) = \frac{4k^3}{\pi^2 M_p^2} |v_k(\tau)|^2. \quad (8)$$

This power spectrum of GWs contributes both temperature fluctuations and polarizations on the CMB.

Both power spectrum and energy spectrum depend on the unknown functions $c_1(k)$, $c_2(k)$. We are particularly interested in the power spectrum at the time $\tau_{\text{obs}} \rightarrow 0$ when the modes exit the horizon and do not evolve anymore. While in general the GWs generated at time τ_* may be either sub-horizon or super-horizon. The relation between them yields:

$$P_\gamma(\tau_{\text{obs}}) = 24H^2 \left(\frac{a(\tau_*)}{k} \right)^2 \frac{k^2 |v_k(\tau_{\text{obs}})|^2}{k^2 |v_k(\tau_*)|^2 + |v'_k(\tau_*)|^2} \Omega_{GW}(\tau_*). \quad (9)$$

We consider the classical limit for these GWs, where $c_1 \approx c_2 \gg 1$. Inserting the mode functions, we can get the relations for sub-horizon and super-horizon modes respectively,

$$P_\gamma(k, \tau_{\text{obs}} \rightarrow 0) = \begin{cases} 24 \left(\frac{a(\tau_*)H}{k} \right)^4 \Omega_{GW}(k, \tau_*), & |k\tau_*| \gg 1 \\ 24 \left(\frac{a(\tau_*)H}{k} \right)^2 \Omega_{GW}(k, \tau_*), & |k\tau_*| \ll 1 \end{cases} \quad (10)$$

In the following section, we are going to mainly discuss the GWs generated by sub-horizon bubbles and leave the super-horizon case to the final discussion.

Gravitational waves by the bubbles. The sub-horizon case in our inflationary scenarios is similar to the previous semi-numerical studies on the EW phase transitions such as [14, 29]. Usually the phase transition is a rapid process compared to the Hubble time and thus the effect of expansion of the universe can be ignored even during inflation. The only difference is that the total energy density ρ_{tot} is much higher with the contribution from the inflaton to drive inflation. Notice that even for sub-horizon bubble, it can stir up turbulence in the thermal plasma and thus generates GWs. For simplicity, we focus on the bubble collision case in the following. Turbulence are expected to have similar effects in spite of more complications [29].

Based on similarity between inflationary and late phase transition for sub-horizon bubbles, one can obtain [14]

$$\Omega_{GW}(k) = \Omega_{GW}^{\text{crit}} \frac{(a+b)k_{\text{crit}}^b k^a}{bk_{\text{crit}}^{a+b} + ak^{a+b}}, \quad (11)$$

$$\Omega_{GW}^{\text{crit}} = \frac{0.11v_b^3}{0.42 + v_b^2} \kappa^2 \left(\frac{H}{\beta} \right)^2 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2. \quad (12)$$

where a, b are exponents parameterizing the scale dependence of the spectrum. Notice that for usual late phase transition we have $\rho_{\text{tot}} \rightarrow \rho_{\text{higgs}} + \rho_{\text{rad}}$, where we can recover the standard results in the literature.

The critical point k_{crit} is also the peak momentum of energy spectrum, which is given by $k_{\text{crit}}/[2\pi a(\tau_*)\beta] = 0.62/(1.8 - 0.1v_b + v_b^2)$, where the bubble wall velocity $v_b \approx 1$ in our discussion here for simplicity and we take $a \approx 2.8, b \approx 1.0$ as in [14]. The parameter β^{-1} is approximately the duration of the phase transition. In sub-horizon case $H/\beta < 1$ although the specific values strongly depend on the shapes of the scalar potentials [30]. And κ characterizes the efficiency of converting vacuum energy into the bubble wall kinetic energy instead of thermal energy. The efficiency factor κ can be determined as a function of $\alpha = \rho_{\text{higgs}}/\rho_{\text{rad}}$, which is the ratio of the Higgs vacuum energy density and the radiation energy density.

We can then finally determine the power spectrum of GW generated by bubbles. Using Eq. (10), we arrive at:

$$P_\gamma(k, \tau_{\text{obs}} \rightarrow 0) = P_\gamma^{\text{crit}} \left(\frac{k_{\text{crit}}}{k} \right)^4 \frac{(a+b)k_{\text{crit}}^b k^a}{bk_{\text{crit}}^{a+b} + ak^{a+b}}, \quad (13)$$

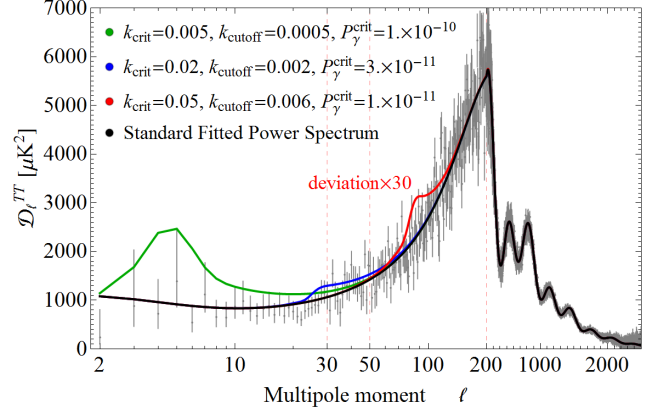


FIG. 3. CMB temperature fluctuation power spectrum. The gray points and error bars are the experimental data of *Planck* 2015 while the black curve is the fitted CMB spectrum using the *Planck* 2015 parameters. Other curves are the corresponding power spectra including GWs generated by bubbles in our scenario. Note that for the red curve, its deviation from the standard one is magnified by 30 times (The unit of the momentum k is Mpc^{-1}).

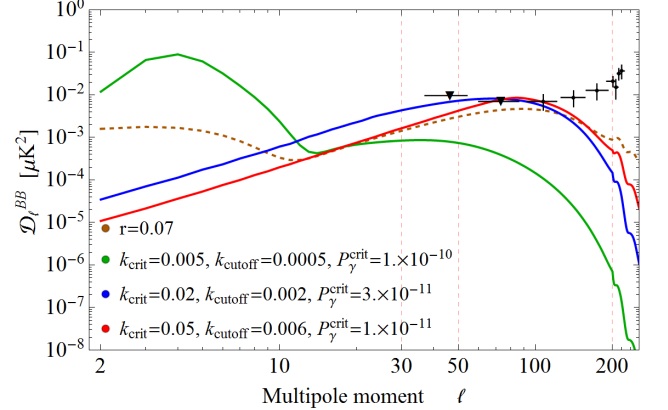


FIG. 4. B-mode power spectrum generated by phase transition bubbles. For comparison, the primordial GWs caused by quantum vacuum fluctuations are shown in dashed line with tensor-to-scalar ratio $r = 0.07$, which is the strongest upper bound to date on primordial GWs. The black ones are the CMB component bandpowers obtained from BICEP2 & *Keck Array* experiment, where error bars denote 68% credible intervals and downward triangles indicate the 95% upper bound [22] (The unit of the momentum k is Mpc^{-1}).

where P_γ^{crit} is the power spectrum at the critical point: $P_\gamma^{\text{crit}} = 24 \left(\frac{a(\tau_*)H}{k_{\text{crit}}} \right)^4 \Omega_{GW}^{\text{crit}}$. An estimation yields: $P_\gamma^{\text{crit}} \sim \left(\frac{H}{\beta} \right)^6 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2$ for the sub-horizon case. The power spectrum is scale dependent and provides us a way to probe phase transition parameters.

Imprints in the CMB. The CMB spectrum can be obtained by inputting the power spectrum into the

CLASS [31] where the transfer function is calculated. As we see from Eq. (13), the power spectrum diverges when $k \rightarrow 0$. This divergence is unphysical as the GW generating formulae break down at super horizon scale. We can introduce the horizon scale as a natural cut-off, yielding $k_{\text{cutoff-physical}} = k_{\text{cutoff}}/a_* \sim H$. The critical physical momentum is related to the bubble size via $k_{\text{crit}}/a_* \sim R_b^{-1}$. The corresponding comoving momenta are

$$k_{\text{crit}} \sim \frac{1}{v_b} \frac{\beta}{H} e^{N_*} k_0, \quad k_{\text{cutoff}} \sim e^{N_*} k_0, \quad (14)$$

where N_* is the e-folding number of phase transition counting from the time that the largest mode k_0 exits the horizon. We choose the scale factor today a_0 as one, thus the largest physical mode today is $k_0 = 0.0002 \text{Mpc}^{-1}$ as the inverse of observed universe size. Approximately we can find k_0 corresponding to the position at CMB multipole $\ell_0 \sim 2$. Then we arrive at relations: $\ell_{\text{crit}} \sim 2e^{N_*} \beta / (v_b H)$, $\ell_{\text{cutoff}} \sim 2e^{N_*}$.

We plot the CMB spectrum with the GW contributions: $P_\gamma^{\text{crit}} \sim 10^{-10} - 10^{-11}$ in Fig. 3 and Fig. 4. The phase transition happened at early time of inflation era generates a new peak on CMB temperature spectrum. The generic peak positions are at $\ell < 200$. For $\ell > 200$, GW modes have already entered the horizon at recombination and thus are subject to rapid decay, posing a greater challenge in experiments. This also implies that the observable phase transitions should happen soon after the largest mode exits horizon with $N_* \lesssim 5$. The amplitudes of those peaks encode the information about the energy scale of inflation and the phase transitions.

Fig. 4 shows the tensorial B-mode spectra, which have not been observed yet in experiment. For primordial GWs caused by vacuum fluctuations, B-mode spectra are known to have the recombination peak at $\ell \sim 100$ and the reionization bump at $\ell < 10$. For $10 \lesssim \ell \lesssim 100$, the spectrum roughly scales like ℓ^2 due to the dominant contribution from recombination [32]. For our scenarios, the power spectrums roughly scale like k^{-1} when $k_{\text{cutoff}} < k < k_{\text{crit}}$ and like k^{-5} when $k > k_{\text{crit}}$. Therefore, if $10 \lesssim \ell_{\text{cutoff}} \lesssim 100$ and $\ell_{\text{crit}} \gtrsim 100$, B-mode spectrum will scale like ℓ for $\ell < 100$. Thus we expect a peak to appear near the recombination peak. Note that for $\ell < \ell_{\text{cutoff}}$, the multipole B-mode spectrum is also contributed by high momentum modes due to the projection tail from k space to ℓ space. More detailed analysis will enable us to extract more physical information about inflationary phase transitions from B-mode spectrum.

The examples below illustrate that, the GWs from the first-order phase transitions can happen within wide range scales and leave visible imprints on the CMB.

- GUT scenarios: The GUT phase transition happens around 10^{16} GeV [33]. A strong first-order phase transition can be achieved in generic scenarios [34]. If choosing $(H/\beta)^6 \sim 10^{-10}$, $\rho_{\text{GUT}} \sim (10^{16} \text{ GeV})^4$, and $H \sim 10^{14} \text{ GeV}$, then we have

$P_\gamma^{\text{crit}} \sim 10^{-10}$. Note, the GUT phase transition can only happen before or soon after the begin of inflation, to avoid re-introducing the problem of magnetic monopoles. Note that in this case, the Hubble scale is allowed to be high, and high Hubble scale inflation implies a component of scale invariant GWs originating from the vacuum. If both the vacuum and bubble GWs are detected, we can infer that the PT is at GUT scale instead of EW scale.

- EW scenarios: Typically the EWPT temperature is around $10^2 \text{ GeV} \sim 10^3 \text{ GeV}$, which requires $T_{GH} \sim H < 10^2 \text{ GeV}$. A first-order EWPT can be achieved in various theories beyond the standard model, e.g. [35–40]. If taking $(H/\beta)^6 \sim 10^{-6}$, $\rho_{\text{higgs}} \sim (10^3 \text{ GeV})^4$, and $H \sim 10^{-11} \text{ GeV}$, then we have $P_\gamma^{\text{crit}} \sim 10^{-10}$. The baryon asymmetry in the universe today can not be directly connected to the first-order EWPT during inflation, due to inflationary dilution. However, if decoupled from inflaton, the effective Higgs potential is sensitive to the temperature of thermal plasma only, subjecting to a negligible curvature correction of order $\mathcal{O}\left(\frac{H}{\Lambda_{\text{EW}}}\right)$. The EWPT during inflation thus may serve as an approximate mirror image of the one after reheating (EWPT in Fig. 2) where the baryon asymmetry could be generated via EWBG. Therefore, the CMB provides a potential way to test the feasibility of the EWBG, complementary to the collider measurements of Higgs potential and the direct detection of GWs generated during EWPT.

Discussions. In this letter we propose an indirect approach to probe the first-order CPTs, such as the GUT phase transition and the EWPT, by detecting the GWs through the CMB. These GWs are generated during inflation via bubble collisions or by the turbulence caused by bubble motion in the thermal plasma, characterized with a scale-dependent power spectrum. They are different from the primordial GWs produced by vacuum fluctuations whose power spectrum is nearly scale-invariant. We calculate the imprints left by the bubble-generated GWs on the CMB and compare the expected power spectra with the latest experimental data.

The large-scale scalar power spectrum caused by the first-order CPTs during inflation might be suppressed. This is due to the relatively large values of the slow-roll parameter when the thermal radiation is diluted. The further discussion on the density fluctuation depends on the details of the specific inflation models. The GWs and density fluctuations generated by super-horizon bubbles share some features in physics discussed above, which may leave imprints on the CMB as well. It remains interesting to investigate reheating in more details, especially in the EW scenarios. We expect preheating [41, 42] may provide efficient reheating, if inflaton and Higgs fields are decoupled. We leave the detailed study to a future work.

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Appendix

Gravitational Wave Spectrum

The metric of tensor perturbations is

$$ds^2 = a^2(\tau) \left[-d\tau^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right], \quad (15)$$

where the traceless, divergenceless and symmetric tensor fields $h_{ij}(\mathbf{k}, \tau)$ satisfy $h_i^i = 0, \partial_i h_{ij} = 0, h_{ij} = h_{ji}$. Our metric sign convention is $(-, +, +, +)$. And the upper and lower indices are the same or, in another word, raised or lowered with δ_{ij} . For example, $h_i^j = h_{il} \delta^{jl} = h_{ij}$. Also, note that the scale factor in quasi de-Sitter space during inflation is $a(\tau) \approx -1/H\tau$. The action for the tensor perturbation is

$$S = \frac{M_p^2}{8} \int d\tau d^3\mathbf{x} a^2 \left[(h'_{ij})^2 - (\nabla h_{ij})^2 \right] = \frac{M_p^2}{8} \int d\tau \frac{d^3\mathbf{k}}{(2\pi)^3} a^2 \left[h'_{ij}(\mathbf{k}, \tau) h'_{ij}(-\mathbf{k}, \tau) - k^2 h_{ij}(\mathbf{k}, \tau) h_{ij}(-\mathbf{k}, \tau) \right]. \quad (16)$$

We can introduce two polarization tensors

$$\epsilon_{ij}^s(\mathbf{k}), \quad (17)$$

with $s = +, \times$ for two degrees of freedom of gravitational waves. They satisfy the following relations (all repeated Latin indices are summed over from 1 to 3)

$$\epsilon_{ij}^s(\mathbf{k}) = \epsilon_{ji}^s(\mathbf{k}), \quad \sum_i \epsilon_{ii}^s(\mathbf{k}) = \epsilon_{ii}^s(\mathbf{k}) = \epsilon_i^i{}^s(\mathbf{k}) = 0, \quad k^i \epsilon_{ij}^s = 0, \quad (18)$$

as well as the following conjugation relation and normalization condition

$$\epsilon_{ij}^s(\mathbf{k}) = \left(\epsilon_{ij}^s(-\mathbf{k}) \right)^*, \quad \epsilon_{ij}^s(\mathbf{k}) \epsilon_{ij}^{s'}(\mathbf{k}) = 2\delta^{ss'}. \quad (19)$$

So, the tensor perturbation field h can be expressed as

$$h_{ij}(\mathbf{k}) = \frac{\sqrt{2}}{M_p} \left(\epsilon_{ij}^+(\mathbf{k}) \gamma_+(\mathbf{k}, \tau) + \epsilon_{ij}^\times(\mathbf{k}) \gamma_\times(\mathbf{k}, \tau) \right), \quad (20)$$

and we have

$$h_{ij}(\mathbf{k}) h_{ij}(-\mathbf{k}) = \frac{2}{M_p^2} \epsilon_{ij}^s(\mathbf{k}) \gamma_s(\mathbf{k}, \tau) \epsilon_{ij}^{s'}(-\mathbf{k}) \gamma_{s'}(-\mathbf{k}, \tau) = \frac{4}{M_p^2} \sum_{s=+, \times} \gamma_s(\mathbf{k}, \tau) \gamma_s(-\mathbf{k}, \tau). \quad (21)$$

Then, it is easy to express the action in terms of γ fields

$$S = \frac{1}{2} \sum_{s=+, \times} \int d\tau \frac{d^3\mathbf{k}}{(2\pi)^3} a^2 \left[\gamma'_s(\mathbf{k}, \tau) \gamma'_s(-\mathbf{k}, \tau) - k^2 \gamma_s(\mathbf{k}, \tau) \gamma_s(-\mathbf{k}, \tau) \right] \quad (22)$$

$$= \sum_{s=+, \times} \int dt \frac{d^3\mathbf{k}}{(2\pi)^3} a^3 \left[\frac{1}{2} \dot{\gamma}_s(\mathbf{k}, t) \dot{\gamma}_s(-\mathbf{k}, t) - \frac{k^2}{2a^2} \gamma_s(\mathbf{k}, t) \gamma_s(-\mathbf{k}, t) \right]. \quad (23)$$

This action is just the sum of two independent copies of spectator scalar fields with $\sigma = \gamma_+, \gamma_\times$,

$$S_\sigma = \int dt d^3\mathbf{x} a^3 \left[\frac{1}{2} \dot{\sigma}(\mathbf{x}, t)^2 - \frac{1}{2a^2} (\nabla \sigma(\mathbf{x}, t))^2 \right] = \int dt \frac{d^3\mathbf{k}}{(2\pi)^3} a^3 \left[\frac{1}{2} \dot{\sigma}(\mathbf{k}, t) \dot{\sigma}(-\mathbf{k}, t) - \frac{k^2}{2a^2} \sigma(\mathbf{k}, t) \sigma(-\mathbf{k}, t) \right]. \quad (24)$$

The classical equation of motion can be obtained by minimizing the action

$$\ddot{\sigma}_\mathbf{k} + 3H\dot{\sigma}_\mathbf{k} + \frac{k^2}{a^2} \sigma_\mathbf{k} = 0. \quad (25)$$

The field can be quantized as

$$\sigma_{\mathbf{k}}(t) = v_k(t)a_{\mathbf{k}} + v_k^* a_{-\mathbf{k}}^\dagger, \quad (26)$$

with the following commutation relations

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}] = [a_{\mathbf{k}}^\dagger, a_{\mathbf{k}'}^\dagger] = 0, \quad [a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = (2\pi^3)\delta^{(3)}(\mathbf{k} - \mathbf{k}'). \quad (27)$$

The mode function is given by

$$v_k(\tau) = \frac{H}{\sqrt{2k^3}} \left(c_1(k)(1 + ik\tau)e^{-ik\tau} + c_2(k)(1 - ik\tau)e^{ik\tau} \right), \quad (28)$$

where the coefficients $c_1(k), c_2(k)$ are subject to the following condition for consistency of quantization $|c_1|^2 - |c_2|^2 = 1$.

The physical energy density is

$$\begin{aligned} \hat{\rho}_\sigma &= \mathcal{H}/a^3 = \frac{1}{2}\dot{\sigma}(\mathbf{x}, t)^2 + \frac{1}{2a^2}(\nabla\sigma(\mathbf{x}, t))^2 \\ &= \frac{1}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{d^3\mathbf{k}'}{(2\pi)^3} e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}} \left(\dot{\sigma}_{\mathbf{k}}\dot{\sigma}_{\mathbf{k}'} + \frac{1}{a^2} i\mathbf{k}i\mathbf{k}'\sigma_{\mathbf{k}}\sigma_{\mathbf{k}'} \right) \end{aligned} \quad (29)$$

Note $\sigma_{\mathbf{k}}\sigma_{\mathbf{k}'} = (v_k a_{\mathbf{k}} + v_k^* a_{-\mathbf{k}}^\dagger)(v_{k'} a_{\mathbf{k}'} + v_{k'}^* a_{-\mathbf{k}'}^\dagger)$, so its expectation value is

$$\langle \sigma_{\mathbf{k}}\sigma_{\mathbf{k}'} \rangle = v_k v_{k'}^* (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}'). \quad (30)$$

Thus, the energy density is

$$\rho_\sigma = \langle \hat{\rho}_\sigma \rangle_0 = \frac{1}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{d^3\mathbf{k}'}{(2\pi)^3} e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}} \left(\langle \dot{\sigma}_{\mathbf{k}}\dot{\sigma}_{\mathbf{k}'} \rangle + \langle \frac{1}{a^2} i\mathbf{k}i\mathbf{k}'\sigma_{\mathbf{k}}\sigma_{\mathbf{k}'} \rangle \right) \quad (31)$$

$$= \frac{1}{2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\dot{v}_k \dot{v}_k^* + \frac{k^2}{a^2} v_k v_k^* \right) \quad (32)$$

$$= \int \frac{dk}{k} \frac{k^3}{4\pi^2} \left(\dot{v}_k \dot{v}_k^* + \frac{k^2}{a^2} v_k v_k^* \right). \quad (33)$$

For gravitational waves, all are the same except the factor 2 due to two polarizations (we assume that the coefficient c_1, c_2 are identical for two polarizations)

$$\rho_{GW} = \rho_+ + \rho_\times = \int \frac{dk}{k} \frac{k^3}{2\pi^2} \left(\dot{v}_k \dot{v}_k^* + \frac{k^2}{a^2} v_k v_k^* \right). \quad (34)$$

The energy spectrum is defined as

$$\Omega_{GW}(k, t) = \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{GW}(t)}{d \log k}. \quad (35)$$

As we assume that the space-time is spatially flat, meaning that $\rho_{\text{tot}} = \rho_{\text{critical}} = 3H^2 M_p^2$. So, finally, we have

$$\Omega_{GW}(k, \tau) = \frac{1}{3H^2 M_p^2} \frac{k^3}{2\pi^2} \frac{|v'_k(\tau)|^2 + k^2 |v_k(\tau)|^2}{a(\tau)^2}. \quad (36)$$

In the sub-horizon limit $|k\tau| \gg 1$, we have the relation $|v'_k|^2 = k^2 |v_k|^2$ (We neglect the interference term between BD-mode and non-BD mode due to rapid oscillations which will become zero after averaging). So, in such case, we have

$$\Omega_{GW}(k, \tau) = \frac{1}{3H^2 M_p^2} \frac{k^3}{\pi^2} \frac{k^2}{a^2} |v_k(\tau)|^2. \quad (37)$$

Inserting the expression for v_k , we obtain (again we neglect the interference term)

$$\Omega_{GW}(k, \tau) = \frac{H^2}{6\pi^2 M_p^2} k^4 \tau^4 (|c_1(k)|^2 + |c_2(k)|^2) = \frac{1}{6\pi^2 H^2 M_p^2} \left(\frac{k}{a}\right)^4 (|c_1(k)|^2 + |c_2(k)|^2) . \quad (38)$$

While in the super-horizon case $|k\tau| \ll 1$, similarly we can get

$$\Omega_{GW}(k, \tau) = \frac{H^2}{12\pi^2 M_p^2} k^2 \tau^2 \left| c_1(k) + c_2(k) \right|^2 . \quad (39)$$

Next, we are going to calculate the power spectrum of the gravitational wave which is defined as

$$\bar{h}^{il} \bar{h}^{jm} \langle h_{ij}(\mathbf{k}) h_{lm}(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} P_\gamma , \quad (40)$$

where $\bar{h}^{ij} = \delta^{ij}$ is the background metric. More explicitly,

$$\bar{h}^{il} \bar{h}^{jm} \langle h_{ij}(\mathbf{k}) h_{lm}(\mathbf{k}') \rangle = \langle h_{ij}(\mathbf{k}) h_{ij}(\mathbf{k}') \rangle = \frac{2}{M_p^2} \epsilon_{ij}^s(\mathbf{k}) \epsilon_{ij}^{s'}(\mathbf{k}') \langle \gamma_s(\mathbf{k}, \tau) \gamma_{s'}(\mathbf{k}', \tau) \rangle \quad (41)$$

$$= \frac{2}{M_p^2} \epsilon_{ij}^s(\mathbf{k}) \left(\epsilon_{ij}^{s'}(-\mathbf{k}') \right)^* |v_k(\tau)|^2 \delta_{ss'} (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \quad (42)$$

$$= \frac{8}{M_p^2} (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') |v_k(\tau)|^2 . \quad (43)$$

So, the tensor power spectrum is

$$P_\gamma(\tau) = \frac{4k^3}{\pi^2 M_p^2} |v_k(\tau)|^2 = \frac{2H^2}{\pi^2 M_p^2} \left| c_1(1 + ik\tau) + c_2(1 - ik\tau) e^{2ik\tau} \right|^2 . \quad (44)$$

If $c_1 = 1, c_2 = 0$, we recover the usual tensor power spectrum $P_\gamma(\tau_{obs} \rightarrow 0) = \frac{2H^2}{\pi^2 M_p^2}$. In the sub-horizon limit,

$$P_\gamma(\tau) = \frac{2H^2}{\pi^2 M_p^2} k^2 \tau^2 \left(|c_1|^2 + |c_2|^2 \right) , \quad (45)$$

while in the super-horizon limit,

$$P_\gamma(\tau) = \frac{2H^2}{\pi^2 M_p^2} \left| c_1 + c_2 \right|^2 . \quad (46)$$

Compare the expression of Ω_{GW} and P_γ , we can get instant relationship between them

$$P_\gamma(\tau) = 24H^2 \left(\frac{a(\tau)}{k} \right)^2 \left(1 + \frac{|v'_k(\tau)|^2}{k^2 |v_k(\tau)|^2} \right)^{-1} \Omega_{GW}(\tau) . \quad (47)$$

In the sub-horizon limit, we have $|v'_k|^2 = k^2 |v_k|^2$, so

$$P_\gamma(\tau) = 12 \frac{a(\tau)^2 H^2}{k^2} \Omega_{GW}(\tau) . \quad (48)$$

In the super-horizon case, all the mode are frozen $|v'_k|^2 / (k^2 |v_k|^2) \ll 1$, so we have

$$P_\gamma(\tau) = 24 \frac{a(\tau)^2 H^2}{k^2} \Omega_{GW}(\tau) . \quad (49)$$

But in practice, we are more interested in the power spectrum in the super-horizon limit where the fluctuations are frozen. We want to connect this frozen power spectrum $P_\gamma(\tau_{obs} \rightarrow 0)$ with the energy spectrum $\Omega(\tau_*)$ at generating time τ_* . Comparing two equations for them, we arrive at the following relation

$$P_\gamma(\tau_{obs}) = 24H^2 \left(\frac{a(\tau_*)}{k} \right)^2 \frac{k^2 |v_k(\tau_{obs})|^2}{k^2 |v_k(\tau_*)|^2 + |v'_k(\tau_*)|^2} \Omega_{GW}(\tau_*) . \quad (50)$$

If gravitational waves are generated at sub-horizon case ($|k\tau_*| \gg 1$),

$$P_\gamma(\tau_{\text{obs}} \rightarrow 0) = 24H^2 \left(\frac{a(\tau_*)}{k} \right)^2 \frac{k^2 |c_1 + c_2|^2}{2k^4 \tau_*^2 (|c_1|^2 + |c_2|^2)} \Omega_{GW}(\tau_*) . \quad (51)$$

If gravitational waves are generated at super-horizon case ($|k\tau| \ll 1$),

$$P_\gamma(\tau_{\text{obs}} \rightarrow 0) = 24H^2 \left(\frac{a(\tau_*)}{k} \right)^2 \frac{k^2 |c_1 + c_2|^2}{k^2 |c_1 + c_2|^2 + k^4 \tau_*^2 (|c_1|^2 + |c_2|^2)} \Omega_{GW}(\tau_*) . \quad (52)$$

In the classical limit, we have real valued coefficients $c_1 \simeq c_2 \gg 1$. So the tensor power spectrum $P_\gamma(\tau_{\text{obs}} \rightarrow 0)$ and energy spectrum at generating time τ_* are related by

$$P_\gamma(\tau_{\text{obs}} \rightarrow 0) = \begin{cases} 24H^4 \left(\frac{a(\tau_*)}{k} \right)^4 \Omega_{GW}(\tau_*) = 24 \left(\frac{H}{k_{\text{phy}}} \right)^4 \Omega_{GW}(\tau_*), & |k\tau_*| \gg 1 \\ 24H^2 \left(\frac{a(\tau_*)}{k} \right)^2 \Omega_{GW}(\tau_*) = 24 \left(\frac{H}{k_{\text{phy}}} \right)^2 \Omega_{GW}(\tau_*), & |k\tau_*| \ll 1 \end{cases} . \quad (53)$$

In the following sections, we are going to determine the energy spectrum of gravitational waves. With Ω , we can get power spectrum using the above formula. With the help of CLASS code, we can get the final CMB power spectrum.

Bubble Collisions

During inflation, the energy of the universe consists of three components: inflaton which drives the inflation of the universe, Higgs field which accounts for the phase transition, and radiation which essentially are the relativistic particles and are related with the temperature of the universe

$$\rho_{\text{tot}} = \rho_{\text{inflaton}} + \rho_{\text{higgs}} + \rho_{\text{rad}} , \quad (54)$$

For our convenience, we also define modified quantities:

$$\tilde{\rho}_{\text{tot}} = \rho_{\text{higgs}} + \rho_{\text{rad}} , \quad \tilde{\Omega}_{GW}(k, t) = \frac{1}{\tilde{\rho}_{\text{tot}}(t)} \frac{d\rho_{GW}(k, t)}{d \log k} , \quad (55)$$

These two quantities are defined to match with previous definitions in literature where there are no inflaton. It enable us to use previous results in our case after some moderate modifications. When there is no inflaton, we recover the usual case, $\rho_{\text{tot}} \rightarrow \tilde{\rho}_{\text{tot}}, \Omega_{GW} \rightarrow \tilde{\Omega}_{GW}$.

We can easily know the relationship between Ω and $\tilde{\Omega}$,

$$\Omega_{GW} = \tilde{\Omega}_{GW} \frac{\tilde{\rho}_{\text{tot}}}{\rho_{\text{tot}}} = \tilde{\Omega}_{GW} \frac{\rho_{\text{higgs}} + \rho_{\text{rad}}}{\rho_{\text{inflaton}} + \rho_{\text{higgs}} + \rho_{\text{rad}}} . \quad (56)$$

Sub-horizon bubble:

If the phase transition is rapid, the bubble size will be much smaller than the Hubble radius. During such short time, the expansion of the universe can be ignored and we expect that almost everything are nearly the same as the usual non-inflationary case which has been well studied as mentioned in the main text.

Some relevant quantities are defined as

$$\alpha = \frac{\rho_{\text{higgs}}}{\rho_{\text{rad}}}, \quad \kappa(\alpha) = \frac{\rho_k}{\rho_{\text{higgs}}} = \frac{1}{1 + 0.715\alpha} \left[0.715\alpha + \frac{4}{27} \sqrt{\frac{3\alpha}{2}} \right], \quad v_b(\alpha) = \frac{1/\sqrt{3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha} , \quad (57)$$

where α is the ratio of Higgs field vacuum energy and radiation energy, while κ characterizes the efficiency of converting the Higgs vacuum energy into the bubble wall kinetic energy instead of heating up the plasma inside the bubble. And v_b is the bubble wall velocity.

More explicitly, the energy density is

$$\rho_{\text{higgs}} = \epsilon_* = -V(v(T), T) + T \frac{dV(v(T), T)}{dT} \Big|_{T=T_*}, \quad \rho_{\text{rad}} = \frac{\pi^2}{30} g_* T_*^4 , \quad (58)$$

where g_* is the effective degree of freedom at phase transition time and T_* is the phase transition temperature. For electroweak phase transitions, $g_* = 106.75$, $T_* \sim 100\text{GeV}$.

The gravitational wave generated by bubble collision has the following rough energy spectrum

$$\tilde{\Omega}_{GW}(f) = \tilde{\Omega}_{GW}^{peak} \frac{(a+b)f_{peak}^b f^a}{bf_{peak}^{a+b} + af^{a+b}} \sim \begin{cases} (f/f_{peak})^a & f \ll f_{peak} \\ (f/f_{peak})^{-b} & f \gg f_{peak} \end{cases}. \quad (59)$$

Numerical analysis shows that $a \in [2.66, 2.82]$, $b \in [0.90, 1.19]$ and the best fit is $a \approx 2.8$, $b \approx 1.0$. f_{peak} is the peak frequency of the energy spectrum given by

$$f_{peak}/\beta = \frac{0.62}{1.8 - 0.1v_b + v_b^2}. \quad (60)$$

Note that f_{peak} is the peak frequency of energy spectrum, *not* necessarily the peak frequency of power spectrum. But anyway, this frequency is a critical point and characteristic frequency. We transformed back to momentum k ($k_{phy} = k/a = \omega = 2\pi f$).

$$k_{crit}/a/\beta = k_{crit-phy}/\beta = 2\pi \frac{0.62}{1.8 - 0.1v_b + v_b^2}, \quad (61)$$

$$\tilde{\Omega}_{GW}(k) = \tilde{\Omega}_{GW}^{crit} \frac{(a+b)k_{crit}^b k^a}{bk_{crit}^{a+b} + ak^{a+b}}. \quad (62)$$

$\tilde{\Omega}_{GW}^{crit}$ is the energy spectrum at critical point, given by

$$\tilde{\Omega}_{GW}^{crit} = \tilde{\Omega}_{GW}^{peak} = \frac{0.11v_b^3}{0.42 + v_b^2} \kappa^2 \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \frac{\rho_{higgs} + \rho_{rad}}{\rho_{tot}}. \quad (63)$$

If there is no inflaton as the usual case, the final fraction term vanishes and we recover previous results in the literature. Except the numeric factor of $\mathcal{O}(1)$, the scaling dimension can also be obtained from the following physical arguments.

A simple and physically intuitive derivations of gravitational wave energy generated by bubble collisions: The relevant physical quantities of this process also include: time scale: $\Delta t \sim \beta^{-1}$, bubble size: $R_b \sim v_b/H \cdot (e^{H\Delta t} - 1)$, bubble volume: $V \sim R_b^3$.

Gravitational energy emission power is given by

$$P_{GW} \sim G \langle (\ddot{Q}^{TT})^2 \rangle, \quad (64)$$

where

$$\ddot{Q}^{TT} \sim \frac{\text{mass} \times \text{size}^2}{\text{time scale}^3} \sim \frac{\text{kinetic energy}}{\text{time scale}}, \quad (65)$$

So, we have $P_{GW} \sim G\dot{E}_k^2$. The kinetic energy of bubble walls is $E_k \sim \rho_k R_b^3 \sim \kappa \rho_H R_b^3$, which leads to $P_{GW} \sim G\dot{E}_k^2 \sim G(E_k/\Delta t)^2$.

Thus, the total energy of gravitational wave is

$$E_{GW} \sim P_{GW} \Delta t \sim GE_k^2/\Delta t \sim G(\kappa \rho_{higgs} R_b^3)^2/\Delta t, \quad (66)$$

and energy density is

$$\rho_{GW} \sim E_{GW}/R_b^3 \sim G\kappa^2 \rho_{higgs}^2 R_b^3/\Delta t. \quad (67)$$

Finally, we get

$$\tilde{\Omega}_{GW} \sim \frac{\rho_{GW}}{\rho_{higgs} + \rho_{rad}} \sim \frac{1}{M_p^2} \kappa^2 \rho_{higgs}^2 R_b^3/\Delta t / (\rho_{higgs} + \rho_{rad}). \quad (68)$$

Inserting the expression of sub-horizon bubble size $R_b \sim v_b \Delta t$, we get

$$\tilde{\Omega}_{GW} \sim \frac{H^2}{H^2 M_p^2} \kappa^2 \rho_{higgs}^2 v_b^3 \Delta t^2 / (\rho_{higgs} + \rho_{rad}) \sim \kappa^2 v_b^3 \left(\frac{H}{\beta}\right)^2 \frac{\rho_{higgs}^2}{\rho_{tot}(\rho_{higgs} + \rho_{rad})}, \quad (69)$$

The scaling behavior is coincident with Eq. (63) except an unimportant factor.

Taking into the numeric factor, we finally get the expression of energy spectrum

$$\Omega_{GW}^{\text{crit}} = \tilde{\Omega}_{GW}^{\text{crit}} \frac{\rho_{\text{higgs}} + \rho_{\text{rad}}}{\rho_{\text{tot}}} = \frac{0.11 v_b^3}{0.42 + v_b^2} \kappa^2 \left(\frac{H}{\beta} \right)^2 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2, \quad (70)$$

$$\Omega_{GW}(k) = \Omega_{GW}^{\text{crit}} \frac{(a+b) k_{\text{crit}}^b k^a}{b k_{\text{crit}}^{a+b} + a k^{a+b}}. \quad (71)$$

Notice that for sub-horizon bubbles, they can also generate super-horizon mode gravitational waves. But the amplitude of these super-horizon modes are expected to be suppressed. So, we can just focus on the sub-horizon modes generated by sub-horizon bubbles. The power spectrum is

$$P_\gamma(k, \tau_{\text{obs}} \rightarrow 0) = 24H^4 \left(\frac{a(\tau_*)}{k} \right)^4 \frac{0.11 v_b^3}{0.42 + v_b^2} \kappa^2 \left(\frac{H}{\beta} \right)^2 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2 \frac{(a+b) k_{\text{crit}}^b k^a}{b k_{\text{crit}}^{a+b} + a k^{a+b}} \quad (72)$$

$$= P_\gamma^{\text{crit}} \left(\frac{k_{\text{crit}}}{k} \right)^4 \frac{(a+b) k_{\text{crit}}^b k^a}{b k_{\text{crit}}^{a+b} + a k^{a+b}}, \quad (73)$$

where P_γ^{crit} is the power spectrum at critical point

$$P_\gamma^{\text{crit}} = P_\gamma(k_{\text{crit}}) = 24H^4 \left(\frac{a(\tau_*)}{k_{\text{crit}}} \right)^4 \frac{0.11 v_b^3}{0.42 + v_b^2} \kappa^2 \left(\frac{H}{\beta} \right)^2 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2 \quad (74)$$

A rough estimation shows that

$$P_\gamma^{\text{crit}} \sim \left(\frac{H}{\beta} \right)^6 \left(\frac{\rho_{\text{higgs}}}{\rho_{\text{tot}}} \right)^2. \quad (75)$$

If we choose $H/\beta = 10^{-2}$, the prefactor is 10^{-12} .

Super-horizon bubble: In some models, the phase transition can last relatively longer. The expansion of the universe can stretch the bubble exponentially. Thus the bubble size can even be larger than the horizon size. Therefore the amplitude of gravitational waves generated by these super-horizon bubbles may be improved dramatically. The exact analyses are very involved.

CMB Imprints

The comoving Hubble radius are the same at present and at the specific point during inflation, corresponding to the horizon exit of the largest mode

$$\frac{1}{a_0 H_0} = \frac{1}{a_i H}. \quad (76)$$

The characteristic physical wavelength is just the physical bubble size, so the characteristic comoving momentum of gravitational waves is

$$1/R_b \cdot a_* = 1/R_b \cdot a_i e^{H(t_* - t_i)} = 1/R_b \cdot \frac{a_0 H_0}{H} e^{H(t_* - t_i)}. \quad (77)$$

The exponential factor characterizes the e-folding-number after the largest mode exiting horizon $N_* = H(t_* - t_i)$. The bubble radius can be estimated as $R_b \sim v_b/\beta$. So, the characteristic or critical comoving momentum is

$$k_{\text{crit}} \sim \frac{1}{v_b} \cdot \frac{\beta}{H} \cdot e^{N_*} \cdot a_0 H_0. \quad (78)$$

Note that the largest mode is just the inverse of the observed universe. If we choose the scale factor today a_0 to be 1, then the largest mode is $k_0 = a_0 H_0 = 0.0002 \text{ Mpc}^{-1}$, corresponding to $\ell \sim 2$ in CMB multipole power spectrum.

$$k_{\text{crit}} \sim \frac{1}{v_b} \cdot \frac{\beta}{H} \cdot e^{N_*} \cdot k_0, \quad \ell_{\text{crit}} \sim 2 \cdot \frac{1}{v_b} \cdot \frac{\beta}{H} \cdot e^{N_*}, \quad (79)$$

In addition, we choose a low momentum cut-off of energy spectrum which corresponds to the Hubble radius

$$k_{\text{cutoff}} \sim e^{N_*} \cdot k_0, \quad \ell_{\text{cutoff}} \sim 2 \cdot e^{N_*}. \quad (80)$$